Qualifying Exam Syllabus

Christopher A. Wong

29 April 2013

Committee: Sanjay Govindjee, John Strain, Jon Wilkening (chair), Maciej Zworski

1 Partial Differential Equations

Classical solutions. Classical solutions to the Laplace, Poisson, heat, and wave equations: fundamental solutions, mean-value properties, existence and uniqueness via energy methods and maximum principles, regularity. Some nonlinear first-order equations: the method of characteristics, Hamilton-Jacobi equations, the Legendre transform and Hopf-Lax formula. ([2] §2.2 – 2.4, 3.2, 3.3.1, 3.3.2)

Sobolev spaces. The general theory of Sobolev spaces: weak/distributional derivatives, the Fourier transform, approximation by smooth functions, extensions, trace operators, Sobolev inequalities and embedding. ([2] §4.3.1, 5.1 – 5.9, [4] §3.1, 3.3, 7.1)

Second-order elliptic equations. Existence, uniqueness, and regularity of second-order elliptic equations, maximum principles. ([2] §6.1 – 6.4)

Linear evolution equations. Weak formulations for second-order parabolic and hyperbolic equations, the Galerkin method, semigroups of linear operators, Hille-Yosida theorem. ([2] §7.1.1, 7.1.2, 7.2.1, 7.2.2, 7.4)

Calculus of variations. First and second variations, the Euler-Lagrange equations, existence of minimizers, weak solutions, Noether's theorem. ([2] §8.1.1 – 8.1.3, 8.2, 8.6)

2 Numerical Linear Algebra

Linear systems. Direct methods for solving linear systems: Gaussian elimination, pivoting, stability, Cholesky factorization, iterative refinement in mixed precision. ([1] §2.1 – 2.5, 2.7.1, [6] §4, 5, 12 – 17, 20 – 23, [3])

Least squares. QR factorization, singular value decomposition, Gram-Schmidt orthogonalization, Householder reflections, stability. ([1] §3.1 – 3.6, [6] §6 – 11, 18, 19)

Eigenvalue problems. Direct methods for solving the symmetric and nonsymmetric eigenvalue-eigenvector problem: Reduction to Hessenberg and tridiagonal form, the power method, bisection with inverse iteration, Rayleigh quotient iteration, the QR algorithm, shifting, divide-and-conquer. ([1] §4.1 – 4.4, 5.1 – 5.4, [6] §24 – 31)

Iterative methods. Iterative methods for solving linear systems and the eigenvalue problem: Jacobi's method, Gauss-Seidel, successive overrelaxation, Arnoldi and Lanczos iteration, conjugate gradient, GMRES. ([1] §6.5 – 6.6, 7.3 – 7.5, [6] §32 – 38)

3 Minor Topic: Functional Analysis

Topological vector spaces. Local convexity, norms and seminorms, completeness, Hahn-Banach theorem, consequences of Baire Category, dual spaces, weak and weak* topologies, Alaoglu's theorem. ([5] §3, 8.1, 8.2, 10, 12, 15)

Linear maps, operators. Spectral theorem for bounded self-adjoint and normal operators, basic properties of compact operators and Fredholm operators. ([5] §21, 31)

Banach algebras. The spectrum, the Gelfand transform, functional calculus. ([5] §17, 18)

References

[1] James W. Demmel, *Applied numerical linear algebra*, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1997.

[2] Lawrence C. Evans, *Partial differential equations*, second ed., Graduate Studies in Mathematics, vol. 19, American Mathematical Society, Providence, RI, 2010.

[3] Nicholas J. Higham, *Accuracy and stability of numerical algorithms*, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1996.

[4] Lars Hörmander, *The analysis of linear partial differential operators. I*, Classics in Mathematics, Springer-Verlag, Berlin, 2003.

[5] Peter D. Lax, *Functional analysis*, Pure and Applied Mathematics (New York), Wiley-Interscience [John Wiley & Sons], New York, 2002.

[6] Lloyd N. Trefethen and David Bau, III, *Numerical linear algebra*, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1997.